



**Figure S5: Effects of exponential cutoffs on the Heaps' law.** All the plots are generated by the stochastic model with  $\alpha = 1$  (i.e.,  $\beta = 2$ ). Instead of a perfect power-law distribution, here the probability density function  $p(z)$  obeys the form  $p(z) \sim z^{-2} \exp(-z/z_c)$ . The first plot in the first line corresponds to the case without cutoff (i.e.,  $z_c \rightarrow \infty$ ). The following four lines correspond to  $z_c = 1000$ ,  $z_c = 100$ ,  $z_c = 10$  and  $z_c = 1$ , respectively. Clearly, as the decreasing of  $z_c$  (i.e., the enhancing of cutoff effect), the Heaps' law still holds with an increasing exponent. The last line shows the result for purely exponential distribution  $p(z) \sim \exp(-z/10)$ . Although this distribution has heavier tail than the one with  $z_c = 1$ , it does not obey the Heaps' law. Actually, it grows almost linearly in the blue area (the fitting slope of the blue area is 0.972) but soon bends.